DETERMINATION OF HEAT LIBERATION COEFFICIENTS WITHIN A CHANNEL BY SOLUTION OF THE CONVERSE THERMAL CONDUCTIVITY PROBLEM

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A method is presented for determination of local heat liberation coefficients, based on numerical solution of the two-dimensional converse thermal conductivity problem.

Among methods for determining nonsteady-state boundary conditions, the most developed are those methods based on solution of one-dimensional converse thermal conductivty problems for thermal sensors of canonical form. However, use of one-dimensional thermal sensors often proves impossible. Therefore, to determine local heat liberation coefficients along a channel length, it is desirable to consider a model channel as a two-dimensional cylindrical thermal sensor.

We will consider a problem of two-dimensional thermal conductivity of a hollow cylinder with nonlinearity of the first sort

$$c(T)\rho\frac{\partial T}{\partial \tau}=\frac{1}{R}\frac{\partial}{\partial R}\left(R\lambda(T)\frac{\partial T}{\partial R}\right)+\frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right), \tag{1}$$

$$0<x<L,\ R_{\mathrm{I}}<R<R_{\mathrm{E}},\ \tau>0$$

with the following boundary conditions:

$$T(x,\ R,\ 0)=T_0,\ 0\leqslant x\leqslant L,\ R_{\mathrm{I}}\leqslant R\leqslant R_{\mathrm{E}};$$

$$T(x,\ R_{\mathrm{E}},\ \tau)=f(x,\ \tau);$$

$$\frac{\partial T(x,\ R_{\mathrm{E}},\ \tau)}{\partial R}=\frac{\partial T(0,\ R,\ \tau)}{\partial x}=\frac{\partial T(L,\ R,\ \tau)}{\partial x}=0; \tag{2}$$

$$-\lambda(T(x,\ R_{\mathrm{I}},\ \tau))\frac{\partial T(x,\ R_{\mathrm{I}},\ \tau)}{\partial R}=\alpha(x,\ \tau)(T_w(x,\ \tau)-T_f(x,\ \tau));$$

$c(T)$, $\lambda(T)$, $f(x,\ \tau)$, $T_f(x,\ \tau)$ are known functions.

Within this formulation we must solve the limiting converse thermal conductivity problem of determining the heat liberation coefficients $\alpha(x,\ \tau)$.

Quantizing the thermal sensor volume leads to the heat measurement system shown in Fig. 1. If we write the thermal balance equation for each of the elements, then with consideration of Eq. (2) we obtain a system of nonlinear algebraic equations:

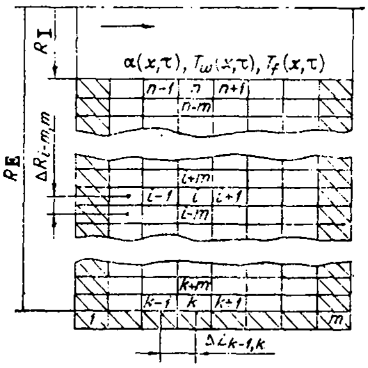

Fig. 1. Heat measurement system (thermal sensor quantization).

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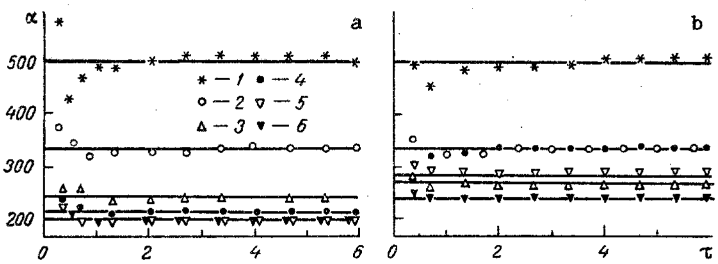

Fig. 2. Numerical experiment on identification of heat liberation coefficient at various distances x/d from input to channel for monotonic distribution of heat liberation coefficient over channel length (a) and for distribution characteristic of turbulent flow at channel input section (b): 1) x/d = 1.67; 2) 5; 3) 8.33; 4) 11.67; 5) 15; 6) 18.33. α, $W/(m^2 \cdot K)$; τ, sec.

for elements on the heat sensing surface

$$\frac{c_n^j \rho V_n}{\Delta\tau^j}(T_n^{j-1} - T_n^j) = \frac{\lambda^j_{w,n} F_{w,n}}{\Delta R_{w,n}}(T_n^j - T^j_{w_n}) + \tag{3}$$

$$+ \frac{\lambda^j_{n-1,n} F_{n-1,n}}{\Delta L_{n-1,n}}(T_n^j - T_{n-1}^j) + \frac{\lambda^j_{n,n+1} F_{n,n+1}}{\Delta L_{n,n+1}}(T_n^j - T_{n+1}^j) + \frac{\lambda^j_{n,n-m} F_{n,n-m}}{\Delta R_{n,n-m}}(T_n^j - T_{n-m}^j),$$

for internal elements

$$\frac{c_i^j \rho V_i}{\Delta\tau^j}(T_i^{j-1} - T_i^j) = \frac{\lambda^j_{i,i-m} F_{i,i-m}}{\Delta R_{i,i-m}}(T_i^j - T_{i-m}^j) + \frac{\lambda^j_{i,i+m} F_{i,i+m}}{\Delta R_{i,i+m}} \times$$

$$\times (T_i^j - T_{i+m}^j) + \frac{\lambda^j_{i,i-1} F_{i,i-1}}{\Delta L_{i,i-1}}(T_i^j - T_{i-1}^j) + \frac{\lambda^j_{i,i+1} F_{i,i+1}}{\Delta L_{i,i+1}}(T_i^j - T_{i+1}^j),$$

for elements on the thermally insulated cylindrical surface

$$\frac{c_k^j \rho V_k}{\Delta\tau^j}(T_k^{j-1} - T_k^j) = \frac{\lambda^j_{k,k+m} F_{k,k+m}}{\Delta R_{k,k+m}}(T_k^j - T_{k+m}^j) +$$

$$+ \frac{\lambda^j_{k,k-1} F_{k,k-1}}{\Delta L_{k,k-1}}(T_k^j - T_{k-1}^j) + \frac{\lambda^j_{k,k+1} F_{k,k+1}}{\Delta L_{k,k+1}}(T_k^j - T_{k+1}^j),$$

where

$$\lambda^j_{i,i+m} = \lambda_0 (1 + b_\lambda (T_i^j + T_{i+m}^j)/2), \qquad c_i^j = c_0 (1 + b_c T_i^j).$$

To determine boundary conditions in cavities with difficult access for thermosensors it is desirable to imbed thermoconverters on the outer thermally insulated surface of the thermosensor (Fig. 1).

Solution of system (3) defines temperatures of the elements and the heat sensing surface for a given moment of time. The next step in solving the converse thermal conductivity problem is determination of the unknown values of specific thermal flux $q_w(x, \tau)$ on the heat accepting surface from the temperature gradient in the adjacent surface layer. The nonlinearity of the first sort is considered by iteration. The heat liberation coefficient on the heat sensing surface is calculated with the expression

$$\alpha(x, \tau) = \frac{q_w(x, \tau)}{T_w(x, \tau) - T_f(x, \tau)}, \tag{4}$$

where

$$T_f(n+1, \tau) = T_f(n, \tau) + \frac{\alpha(n, \tau) F_w (T_w(n, \tau) - T_f(n, \tau))}{G(\tau) C_p}. \tag{5}$$

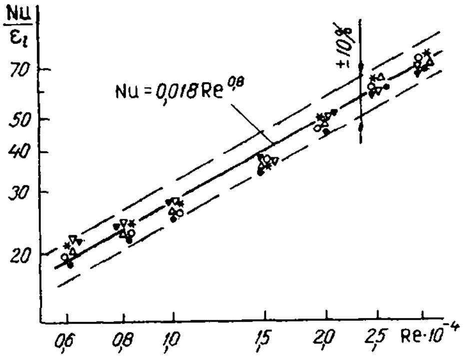

Fig. 3. Results of experiments to test heat liberation coefficient determination technique. Notation as in Fig. 2.

The problem is regularized by using spline-smoothing of the temperature sensor indications according to the method proposed in [1] with an algorithm for automatic search for weight coefficients [2].

Using the method described above a program was written in the Fortran-IV language for an ES computer. To test the method of identifying heat liberation coefficients in the presence of noise in the initial data a numerical experiment was performed for a cylindrical thermal sensor of 12Kh18N10T steel with dimensions as follows: internal radius R_I = 0.0015 m, external radius R_E = 0.0035 m, length L = 0.06 m with monotonic distribution of the heat liberation coefficient along the channel length and for the distribution characteristic of a turbulent flow at the channel input section.

The comparison of prespecified and reconstructed heat liberation coefficients shown in Fig. 2 indicates that the method of determining $\alpha(x, \tau)$ by solution of the two-dimensional converse thermal conductivity problem produces stable results over the entire time interval except for the initial period up to $\tau < 0.8$ sec. Deviation of the $\alpha(x, \tau)$ values obtained from the prespecified ones did not exceed 5% for $\tau \geq 0.8$ sec.

Experimental testing of the proposed method was carried out by a study of heat liberation in a circular tube using the apparatus described in [3].

The working section was a cylindrical tube ($D = 7 \cdot 10^{-3}$ m, $d = 3 \cdot 10^{-3}$ m) with relative length L/d = 20, constructed of 12Kh18N10T steel. In accordance with the proposed method six uniformly spaced type KhK thermocouples with electrode diameter $0.2 \cdot 10^{-3}$ m were welded to the outer channel surface. At the attachment points the thermoelectrodes were flattened to a thickness of $(0.4\text{-}0.5) \cdot 10^{-4}$ m. The channel was thermally insulated on the outside by cotton-paper wool and asbestos, and on its faces by paper 0.5-0.7 mm thick.

Results of numerical experiments performed on an ES-1033 computer with a Fortran-IV program are shown in Fig. 3, whence it is evident that within an accuracy of ±10% the experimental points coincide with the known dependence for heat exchange in turbulent flow of air in a circular tube [4]:

$$\mathrm{Nu} = 0{,}018\,\mathrm{Re}^{0,8}\,\varepsilon_l, \tag{6}$$

where ε_ℓ is the correction for the initial segment.

It should be noted that the need to find steady state heat liberation coefficients imposes limitations on the experiment parameters, since thermal and gas dynamic nonsteady states significantly affect heat liberation [5]. To evaluate this effect similarity numbers were defined:

$$K_{Tg}^{*} = \frac{\partial T_w}{\partial \tau}\,\frac{d}{T_w}\sqrt{\frac{\lambda}{gC_pG}}, \tag{7}$$

$$K_{Gg} = \frac{\partial G}{\partial \tau}\,\frac{d}{G}\sqrt{\frac{\mu}{G}}, \tag{8}$$

characterizing the effect of thermal and gas dynamic nonsteady state conditions on heat exchange. The intervals of change of the numbers K_{Tg}^* and K_{Gg} were as follows:

$$0 \leqslant K_{Tg}^{*} \leqslant 0{,}5 \cdot 10^{-6}, \quad 0 \leqslant K_{Gg} \leqslant 10^{-6}.$$

Based on the results of the experiments performed we may conclude that for the indicated values of the criteria $K_{Tg}*$ and K_{Gg} the nonsteady state does not affect heat liberation [5], which permits us to treat the parameter change modes as quasisteady states.

The results obtained confirm the reliability and effectiveness of the method for determining heat liberation coefficients considered herein.

NOTATION

T, temperature; T_w, temperature of heat-sensing surface; T_f, heat exchange agent temperature; λ, thermal conductivity coefficient; c, specific heat; ρ, density; x, spatial coordinate; R, R_I, R_E, current, internal, and external radii of sensor; L, sensor length; ΔR, distance between approximation points along thermal sensor thickness; ΔL, distance between approximation points along thermal sensor length; V, volume of sensor element; F, area; α, heat liberation coefficient; τ, time; $\Delta\tau$, step in the time; n, i, k, element numbers; j, time step number; λ_0, b_λ, c_0, b_c, coefficients of approximating linear temperature dependences of thermal conductivity coefficient and sensor material specific heat; q_w, thermal flux density on heat sensing wall; μ, dynamic viscosity; G, heat exchange agent flow rate; Cp, specific heat of heat exchange agent; d, channel diameter.

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DETERMINATION OF THERMAL FLUX DENSITY, MEDIUM TEMPERATURE AND HEAT LIBERATION COEFFICIENT BY SOLUTION OF THE CONVERSE THERMAL CONDUCTIVITY PROBLEM

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The converse thermal conductivity problem of determining temperature of the hot medium, heat liberation coefficient, and thermal flux density for asymmetic heating is solved using results of wall temperature measurements at three points located different distances from the hot surface.

In operation of high power equipment experimental determination of the temperature of the hot medium, the thermal flux density, and the heat liberation coefficient from the medium to the body wall under nonsteady state conditions is difficult, since thermal sensors will not tolerate the high thermal loads involved. In connection with this one can solve the converse problem of determining the basic parameters of the nonsteady state heat exchange between the hot medium and the body wall by using measurements of temperature over time at three points located at different distances from the heated (hot) wall surface.

We will consider the one-dimensional process of heat transport within a wall, one surface of which is heated by the hot medium, while the other is cooled by a cold medium in accordance with boundary conditions of the third type.

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